

A MORTALITY MODEL AND ITS IMPLICATIONS FOR THE THAI ELDERLY

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Abstract

The objective of this study aims at modeling the mortality rate of Thai elderly by using the Inverse-Makeham model, together with the Coale-Kisker method. The data used in the study are the size of the Thai population at the end of the year and the number of deaths classified by gender and the age of the Thai population from 2003-2016. The Lee-Carter model was also applied to forecast the mortality rate of the Thai elderly for the next 11 years, from 2017 to 2027, and then to adjust the Mortality Table of Thailand. The results revealed that, when the Lee-Carter Model estimating the parameter using the Singular Value Decomposition was applied, the predicted mortality rates during the years 2017 - 2027 for males and females were in the same direction and are likely to decrease over time. In the meantime, the life expectancy of both sexes has been declining until the age of 60, and the life expectancy then began to decline at a slower rate until approaching 0 at age 110. As a consequence, the study used the results of the analysis to adjust the Thai Mortality Table for the years 2017 and 2027 in order to present which changes rapidly will occur in the structure of the Thai population, which is living longer today. The study recommends that the insurance sector should prepare to handle the changes in the Thai society (more elderly people) and higher longevity risks in the Thai society in order to design appropriate life insurance and insurance products. In the meantime, the government should expedite the findings of the measures to support the increasing number of Thai elderly in the near future, as forecasted in this study.

Keywords: mortality modeling, forecasting, implications, Thai elderly, longevity risk management

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Introduction

The aging of the population in the world is a worldwide phenomenon at the beginning of the millennium. For Thailand, the population is aging very quickly and it will become an absolute aging society in the next 15 years. In the past 50 years, the age structure of the Thai population has dramatically changed, and the Thai population has a similar birth rate and mortality rate each year. This is because people have less successors due to the rapidly changing society and the economy of the country, with the development of technological advances in medicine and public health, resulting in the overall Thai population structure tending to live longer and the mortality rate of the Thai population is continuing to decline. This is causing birth rates to drop and that makes the Thai population as a whole increase at a very low rate, while the elderly population has increased at a very high rate. Hence, the age of the population age is increasing at a fast rate.

Because Thailand is about to enter an aging society, the Thai people, both males and females, tend to live longer than in the past and the mortality rate in the Mortality Table 2008 that is promulgated may not be suitable for current and future events that are constantly changing. The forecasting of mortality is considered important in studying the trends of the population because the death rate can be as important as the birth rate. In the insurance business, actuaries need to be able to create an accurate mortality table in order to determine insurance premiums and premium reserves. The mortality rate is also useful in other disciplines, such as medicine and public health, which use life tables to find the average lifespan and longevity of the population. Nonetheless, forecasting the mortality rate is quite complicated due to a number of constraints, such as natural disasters, new diseases, or other factors such as medical advancement. This study however will not take these factors into consideration; it only forecasts future mortality rates based on the predictions from the statistical trends of past mortality. In the past, death rate predictions have been arrived at in many ways in each region of the world. The problem of this research therefore is to study how to estimate and forecast the mortality rate of Thai elderly to be suitable in accordance with the changing context of Thailand. Additionally, the results obtained from the estimates and forecasts for adjusting the existing Thai Mortality Table to be up to date and in line with current and future situations that change rapidly in the structure of Thai population. As a result, the values from the adjusted Thai mortality table can be applied as a basis for finding insurance premiums for pension life insurance, which pays benefits for the insured from the age of 60 and is truly in line with the current situation. In addition, the study results can be used as basic information in formulating government policies to support Thai society toward an aging society.

Research Objectives

1. To formulate a mortality model for Thailand by using appropriated estimation and forecasting techniques.

2. To forecast the mortality rate of the population of the Thai elderly for the next 11 years, from 2017 to 2027.
3. To apply the formulated mortality model to adjust the Mortality Table 2008 for Thailand.

Literature Review

Population Aging Trends in the World and in Thailand

In recent years, population aging has become a common phenomenon occurring in many countries in the world. This reflects that the world's population has currently entered an aging society. According to the Foundation of Thai Gerontology Research and Development Institute (2018), the world's population is getting bigger. In 2017, it reported that there were 962 million people who aged 60 or older in the world, representing 13 percent of the total population. In addition, the United Nations Population Fund (UNFPA) and HelpAge International have reported concerning the situation of the elderly in the world in the report "Ageing in the Twenty: Celebrations and Challenges" that in the year 1950, the world's population aged 60 years and over was over 205 million, while in 2012 the number of older people had increased to almost 810 million. This number will be more than 1 billion people in less than 10 years and will double by 2050, or over 2 billion elderly people. Each region has a different population in terms of the elderly: for example; in Africa, the population aged 60 and over will be 10 percent, while Asia, North America, and Europe will have an elderly population of more than 24 percent, at 27 and 34 percent respectively. Japan is the country that has entered the most senior society with a proportion of people aged 60 years and over up to 33.1 percent of the total population of the country (United Nations Population Fund: UNFPA, 2012).

For the countries in the ASEAN region, with a total of 10 members, it has been found that all of the countries are tending to enter the aging society. In 1999, ASEAN reported that the region had a population aged 60 years or more over 38 million people, representing 7.3% of their total population in the region. Singapore has also been classified as an aging society. By 2017, the population of ASEAN increased to 647 million and the population almost doubled to 64 million. There are two more ASEAN member countries that have become aging societies, namely Thailand and Vietnam. While Singapore became a "complete aging society," at least 2 other countries, namely Malaysia and the Union of Myanmar, became aging societies by 2019 (Foundation of Thai Gerontology Research and Development Institute, 2018). According to the United Nations, which as forecasted the average expected age between 2010 and 2015, it appears that the average expected age in developed countries will be 78 years, and in developing countries it will be 68 years, and during the years 2045-2050 the United Nations also predicted that new-born babies in developed countries will live up to 83 years and in developing countries the average age will be 74 (United Nations Population Fund: UNFPA, 2018), as shown in Figure 1.

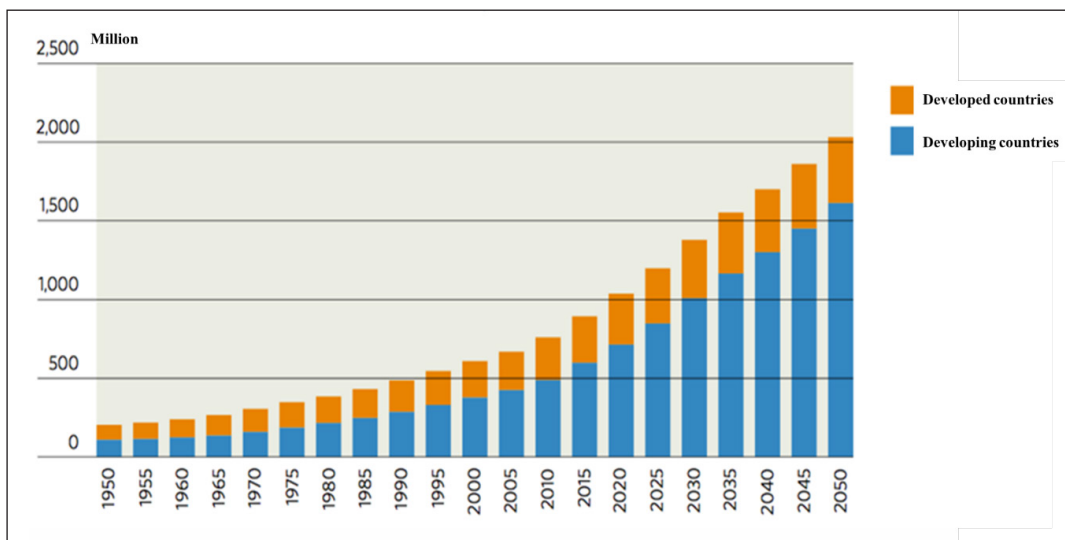


Figure 1 Comparing the population aged 60 and over in developed and developing countries between 2010 and 2015.

Source: United Nations Population Fund (2012).

The Foundation of the Thai Gerontology Research and Development Institute (2018) reported that since 2005, the Thai population has had a proportion of the population aged 60 years and over of up to 10 percent, and Thailand has become an "elderly society" since that year. In 2017, however, the rate of Thai elderly increased to 17% and will become a "complete aging society" during the next decade. It has been currently found that the total number of the newly born population has also increased at a very low rate—close to zero—and is likely that Thailand will have a negative population growth rate soon. On the other hand, the aging population has increased at a very high rate, and the aging population is increasing rapidly. In 1990, Thailand had only over 400,000 senior citizens aged 80 years, but by 2017 the number of senior citizens increased to 1.5 million. In 2017, it was estimated that the senior citizens will be up to 3.5 million people over the next 20 years. From 2017 onwards, it is also estimated that the population aged 60 years and over will increase at an average rate of 4 percent per year, while the population aged 80 years and over will increase at an average rate of 6 percent per year. This information clearly reflects the rising trend of the elderly in Thailand.

In addition, the Bank of Thailand (2018) has reported that Thailand is transitioning from an aging society and is stepping into a complete aging society at a faster rate than other developing countries, where generally the structural change process usually takes 18 to 115 years, while in Thailand it takes only 20 years. Therefore, by 2035, Thailand will be the first developing country to fully enter an aging society. However, the countries that have advanced to being a senior society are developed countries and have an average per capita income of more than US \$12,500 per

person per year, whereas the current per capita income in Thailand is only 5,700 US dollars. As a result, the impact of the population structure changes working decisions, and the ability of the Thai population to earn an income is changing quickly and may be more severe than in many other countries by comparison.

Estimation and Forecasting of Mortality

In estimating mortality rates, scholars from many countries have been studying suitable models and have applied various models to the population in their countries. These efforts have been made to examine different population groups in order to obtain an appropriate model for the study group and applicable mortality rates. For instance, Husted (2005) studied the high mortality rates of the populations in many countries, from the age of about 90 years or more, in order to estimate the mortality rate, which has patterns from using many theories or models such as the Forced Method, the Blended Method, the Pattern Method, and The Least-Than-One Method. Husted's study explores how mortality tables have been ended in the past and addresses the question whether an ending rate of 1.000 is most appropriate. Gavrilov, Gavrilova, and Krut'ko (2017) reported a variety of methods used to formulate mortality rates, which are classified as parametric models and the nonparametric approach. Parametric models began with the introduction of the Gompertz law (1825), but according to further studies, the Gompertz law does not work well for people over 85 years of age. Later, Makeham (1860) added another parameter to the model, which is a parameter regarding age-independent death factors, known as the Gompertz-Makeham law. This law states that the human death rate is the sum of an age-dependent component (the Gompertz function), which increases exponentially with age and an age-independent component (Mumpar-Victoria, Hermosilla and Mirandilla, 2005). There has also been trying to improve the Gompertz function in the form of a logistic equation called the Kannisto model, which provides a better fit with the mortality data of people aged over 85 than Makeham's model, where the force of mortality increases exponentially with age (Doray, 2008)

For the forecasting of mortality rates, many studies have been undertaken in order to find suitable models for population data. For example, Booth and Tickle (2008) described the different models that are used in the prediction of the mortality rate from 1980 to 2008. These models can be classified according to three forecasting types: (i) expectation, which is the principle based on the personal opinions of experts in the various fields involved; (ii) extrapolation or inferential estimates, which is a method that relies on past data with the assumption that future trends will be continuous from the past; and (iii) explanation, which is a method that uses structures or epidemiological models by considering the factors that cause death. The methods of estimation out of range or inference, the extrapolative approach, are the most popular form. The Lee-Carter model is a numerical algorithm used in mortality forecasting and life expectancy forecasting and is one of the methods of estimating out of range or inference (extrapolative approach) (Lee and Carter, 1992). This model is also a model for describing the secular change in mortality as a function of

a single time index, specifying a log-bilinear form for the force of mortality. Subsequently, Dowd et al. (2010) studied and set out a framework to evaluate the goodness of fit of stochastic mortality models and applied it to six different models, which have the main assumption that the residue series for each model is independent and has the same standard normal distribution. Studies have shown that no model is obviously better than other models. The well-tested condition of the residual price of annuity shows that the Lee-Carter model is the most suitable. As for the method of estimation of parameters in the Lee-Carter model, a study in Thailand by Natthasurang Yasungnoen (2015), who has studied the mortality rate of the population of Thailand and Nakhon Ratchasima by using the Lee-Carter model and the age-period-cohort model. The study found that the Single Value Decomposition (SVD) is the best fit for the parameter estimation for females and the Weighted Least Squares (WLS) is the best fit for males.

Research Methodology

Research Design

This research aims to formulate a mortality model for the Thai elderly, taking the results from the estimates and forecasts to adjust the Thai mortality table based on suitable present which is changing rapidly in the structure of the Thai population. The researcher therefore mainly conducted quantitative research, which consists of studying documents and statistical reviews (Desk review) that were currently available and analyzing secondary data from large data sources in order to achieve the set objectives.

Sources and Collection of the Data

This study investigated the goodness of fit for the mortality rate spanning the period 2003 to 2016. The secondary data collected for the mortality rate model of Thai elderly in this research were the number of age-specific people in the population, and the number of deaths was obtained from relevant authorities with details and sources as follows.

(i) The size of the Thai population at the end of the year as of December 31 of each calendar year from 2003 to 2016 from the Department of Provincial Administration, Ministry of Interior (2019), classified by yearly age, under 1 year to over 100 years and over 100 years, and sex

(ii) The number of deaths from 2004 to 2016 from the Bureau of Policy and Strategy, Ministry of Public Health, classified by yearly age, under 1 year to over 100 years and over 100 years, and sex

(iii) The Thai mortality table 2008, for both males and females, announced by the Insurance Commission of Thailand (2009)

Data Analysis

1) Data preparation: Due to the data on the number of deaths and the size of the population collected in each year having different characteristics, data adjustment was necessary before the analysis. By adjusting the number of deaths, converting end-of-year population data to mid-

year population numbers and the determination of the central death rate were carried out.

2) Estimation and forecasting of mortality rate: The target group in this study was the elderly and the Thai Mortality Table 2008 promulgated by the authority may not be suitable for current and future events that have continued changed; therefore, the estimation of the mortality rate for the elderly was computed. There are however many ways to project future mortality rates. According to a study of Sayhumpooh Saichonpitak (2010), it was found that the Inverse-Makeham model is the most suitable for estimating mortality. For forecasting, the Lee-Carter model has proved to provide good results for mortality in diverse countries (Lee and Miller, 2001). This research therefore applied the Inverse-Makeham model and the single factor Lee-Carter model, which are suitable for the aims of the present investigation. The forecasted mid-year mortality rate then was adjusted to the mortality rate under the assumption that the number of deaths in each age range was distributed evenly over the entire period (Uniform-distribution of death, UDD).

3) Use the forecasted mortality rate to adjust the Thai Mortality Table for 2017 and 2027, compared with the table in 2008.

This study used Microsoft Excel 2010 and R version 3.5.3 to calculate and compute the parameters of each model.

Results of the Study

Estimation of Mortality Rate

The results of the mortality rate forecasted for 2017-2027 revealed that the mortality rates for males and females, as shown in Figure 2 and Figure 3 respectively, showed a decreasing trend. Hence, the Thai population is likely to experience a longer lifespan in the future.

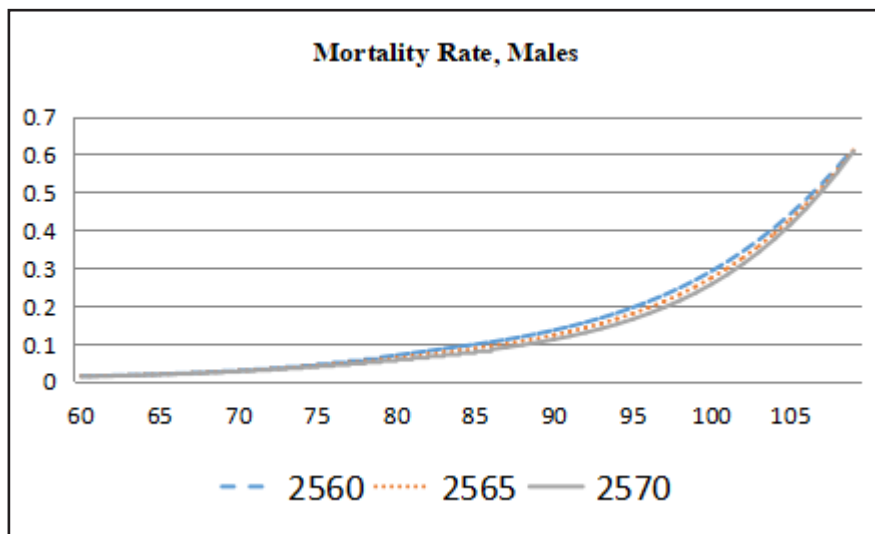


Figure 2 A forecasted mortality rate comparison between 2017 and 2027 for males

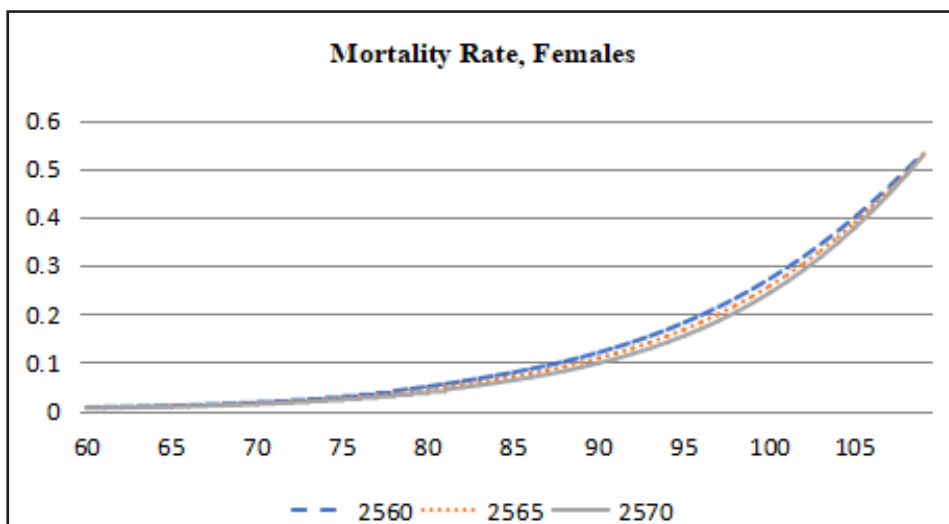


Figure 3 A forecasted mortality rate comparison between 2017 and 2027 for females

According to Figure 2 and Figure 3, it can be observed that the forecasted male mortality rate is higher than that of women of the same age in all age groups. The mortality rate for both sexes however tends to be in the same direction, i.e. the rate will increase steadily and will have a value of 1, which is in accordance with the law of the mortality rate. In addition, when comparing the mortality rates during 2017, 2022, and 2027 in order to observe the changes over time, it can be seen that for both males and females, the forecasted mortality rate in 2027 is lower than the mortality rate in 2022.

Adjusting the Thai Mortality Table

The results of estimating and forecasting the mortality rate of Thai elderly has been applied to adjust the mortality table in Thailand. The new mortality table then resulted in Figure 4 and Figure 5, which showed that the expected age of males and females of different ages would have different values. The average male age was predominantly lower than that of females throughout the age range from 60 to 110 years, and the expected age of both sexes tended to be in the same direction, which decreases until the value converges into 0 when the person is 110 years old.

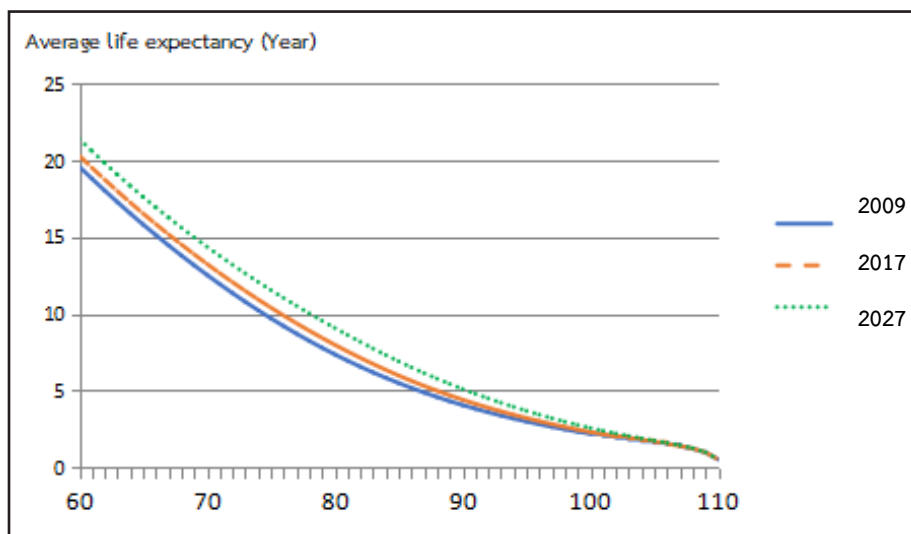


Figure 4 A comparison of the life expectancy age of 60-110 years in the mortality tables for males in 2009, 2017 and 2027

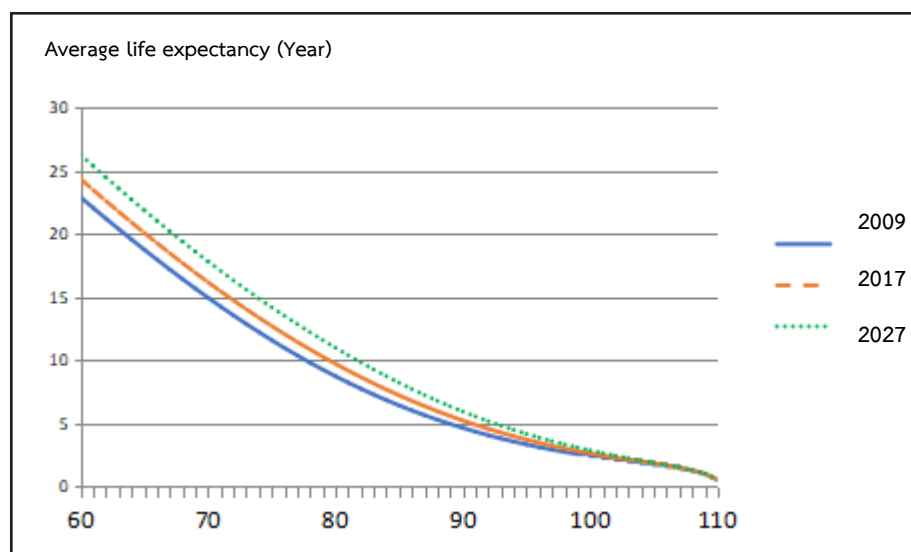


Figure 5 A comparison of the expected age of 60-110 years in the mortality tables for females in 2009, 2017 and 2027.

In addition, when comparing the average expected age in the forecasted Thai mortality tables of 2009, 2017, and 2027 in order to observe changes over time, the results revealed that for both males and females, the average life expectancy for 2027 was higher than the average life expectancy for 2017. Moreover, the average life expectancy for the year 2017 was also higher than the average life expectancy for 2009. This was based on the assumption that the Thai population, both males and females, are living longer than in the past.

Finally, Table 1 and Table 2 showed the Thai mortality table in 2017 and 2027 that had been adjusted by the results obtained from the estimation and forecasting of mortality rates for Thai elderly.

Table 1 Adjusted Mortality Table 2017

Males					Females				
Age (x)	Lives l(x)	Deaths d(x)	Average life expectancy (x)	Mortality rate 1000q(x)	Age (x)	Lives l(x)	Deaths d(x)	Average life expectancy (x)	Mortality rate 1000q(x)
0	1,000,000	4,723	76.966	4.7231	0	1,000,000	2,682	82.554	2.6817
1	995,277	782	76.329	0.7855	1	997,318	387	81.775	0.3881
2	994,495	573	75.388	0.5764	2	996,931	316	80.806	0.3171
3	993,922	419	74.432	0.4220	3	996,615	271	79.832	0.2716
4	993,502	335	73.463	0.3374	4	996,344	239	78.853	0.2397
...
100	12,253	3,944	2.294	321.8946	100	36,520	10,288	2.591	281.7065
101	8,309	2,858	2.145	344.0085	101	26,232	7,935	2.411	302.4890
102	5,451	1,995	2.008	366.0569	102	18,297	5,943	2.239	324.8238
103	3,455	1,343	1.879	388.8163	103	12,354	4,305	2.076	348.4569
104	2,112	871	1.756	412.2192	104	8,049	3,010	1.919	373.8948
105	1,241	541	1.637	436.1867	105	5,040	2,020	1.766	400.8221
106	700	322	1.516	460.6287	106	3,020	1,297	1.613	429.4066
107	377	183	1.383	485.4440	107	1,723	791	1.450	458.9036
108	194	99	1.217	510.5883	108	932	456	1.255	489.4717
109	95	51	0.964	535.8970	109	476	248	0.979	520.6858
110	44	44	0.5	1,000.0000	110	228	228	0.5	1,000.0000

Table 2 Adjusted Mortality Table 2027

Males					Females				
Age (x)	Lives l(x)	Deaths d(x)	Average life expectancy (x)	Mortality rate 1000q(x)	Age (x)	Lives l(x)	Deaths d(x)	Average life expectancy (x)	Mortality rate 1000q(x)
0	1,000,000	6,119	79.002	6.1192	0	1,000,000	2,778	84.968	2.7784
1	993,881	253	78.486	0.2541	1	997,222	125	84.203	0.1255
2	993,628	221	77.505	0.2222	2	997,096	122	83.213	0.1222
3	993,408	184	76.523	0.1849	3	996,975	119	82.224	0.1190
4	993,224	162	75.537	0.1631	4	996,856	116	81.233	0.1159
...
100	25,609	7,272	2.555	283.9617	100	66,177	16,652	2.831	251.6309
101	18,337	5,627	2.370	306.8645	101	49,525	13,527	2.615	273.1442
102	12,710	4,199	2.198	330.3683	102	35,997	10,676	2.410	296.5832
103	8,511	3,023	2.035	355.2041	103	25,321	8,148	2.215	321.7705
104	5,488	2,093	1.881	381.3489	104	17,173	5,997	2.029	349.2260
105	3,395	1,388	1.732	408.7562	105	11,176	4,232	1.849	378.7109
106	2,007	878	1.584	437.3518	106	6,944	2,850	1.671	410.4288
107	1,129	527	1.427	467.0303	107	4,094	1,816	1.487	443.7015
108	602	300	1.239	497.7169	108	2,277	1,090	1.274	478.6917
109	302	160	0.971	529.1925	109	1,187	611	0.985	514.9813
110	142	142	0.5	1000.0000	110	576	576	0.5	1000.0000

Conclusion

This research aimed to create a mortality model for the Thai elderly using the Inverse-Makeham model and the Lee-Carter model for estimating and forecasting the mortality rates in Thailand and taking the results from the estimates and forecasts to adjust the Thai Mortality Table based on suitable present which changed rapidly in the structure of Thai population, which is living longer today. Upon receiving the updated Thai mortality table, it will be basic information for computing life insurance premiums for pension insurance and can be used as a basis for government information in formulating relevant policies to support the aging society of Thailand. The study investigated the goodness of fit for the mortality rate spanning the period 2003 to 2016. The Ministry of the Interior was the main source for data for the number of age-specific people in the

population, while the number of deaths was obtained from the Bureau of Policy and Strategy, Ministry of Public Health of Thailand.

In estimating the mortality rate of the Thai elderly using the Inverse-Makeham model, the mortality rate can only be estimated from the age of 60 years to about 85 years, for both males and females, which was the maximum age (u) that this model could use. For a higher age range, that is, between the ages of 85 to 110 years, the mortality rate must be adjusted using the Coale-Kisker method in order to estimate the mortality rate in compliance with the law of mortality, meaning that as age increases, the mortality rate also increases. Next, the estimated mortality rate was forecasted by using the Lee-Carter model with the Specific Separation Method. When determining the mortality rate using a time series, the mortality rate for both males and females showed a decreasing trend. This trend implies that the Thai population is likely to experience a longer lifespan in the future. As a result of the study, the forecasted mortality rate can be suitably applied to adjust the Thai Mortality table.

Discussion

In forecasting the mortality rate and life expectancy for the Thai population, it can be concluded that the forecasted male and female mortality rate is decreasing in the same direction, where the mortality rate of males is higher than that of females. When considering the forecast of the mortality rate by age, it was found that the predictability of the mortality rate tended to decrease over time, which is the same as reported by many researches, including the research of Lee and Carter (Lee and Carter, 1992), the study of the mortality model of the Thai population by Nicha Racharit and Suwanee Surasiengsunk (2006), and the study of Thai mortality estimation by Nattakorn Suramethakul (2009). These studies found that the male and female mortality predictions were in the same direction, with the mortality rate decreasing slowly. In addition, the mortality rate trends obtained from the model were consistent with the technological advances in medicine and public health in Thailand. These are important factors that have caused a continuous decline in the mortality rate in many parts of the world. The findings are also consistent with the research on mortality and longevity of the Thai population by Pattama Waphatthanawong and Pramot Prasatkun (2006). It can be concluded that now, and in the future, Thai people will live longer than in the past and this is in line with medical and technological advancements. Furthermore, the higher level of education of Thai citizens creates health consciousness regarding healthy foods and emphasized more exercise, which are also important factors for mortality rate reduction. As a result, in the future, Thailand will have a larger elderly population and this will finally affect the pension benefits of Thailand.

Recommendations

1) When compared to the demographic data that has been collected in developed countries, it appears that the secondary data used in this study found that there is a lot of incomplete data, especially the data on the number of unknown deaths. In this study, the researcher adjusted the number of unknown deaths by distributing this information to different ages, which may have caused discrepancies regarding data deviations, resulting in possible errors in the study. The relevant agencies involved in the data collection and registration processes then should pay attention to the completeness of registration and the collection of statistical data so that it is accurate and complete, and they should study the methods of collecting data that meet international standards. This will enable us to forecast the mortality rate more accurately in the future.

2) As a consequence of the incomplete data, this study had abnormal death rates during two periods; (i) the first period is the newborn period, that may be recorded in both the population data and death data; and (ii) the second period is in the late age or elderly period, where the mortality rate dropped sharply, which is inconsistent with reality. Therefore, in future research, the information should be adjusted before studying or the abnormal age range should be further studied. In addition, the adjustments in the various variables studied should consider other factors that may affect the life of the Thai population, for example, daily living habits, smoking, food, eating, various diseases, etc. Hence, the Thai people's behavior should be studied in terms of the important factors that may affect mortality and appropriate methods of adjustment should be sought in order to make the information used in the study more accurate and reliable.

3) This study has time limitations in the data collection while there are many forecasting models for mortality rates. The assumptions of the mortality model and the parameter estimation method have an impact on the forecasted mortality model. In future research, other comprehensive forecasting mortality models and methods for the estimation of parameters, as well as the most up-to-date information with various data validation and improvement methods can be applied to mortality data for the population. This will also enable the studies to forecast the mortality rate more accurately and consistently within the context in Thailand.

4) The results show that it is clear that Thai people will live longer in the future and there will be a tendency for the elderly to increase in number. Thus, this group of Thai people may not be able to happily retire or cope with the burden of living expenses after having retired. The risk that the elderly will have a longer life than the assets called "longevity risks" is a high concern. Longevity risks can, however, be managed to a certain degree by setting and adjusting the underlying investments, asset allocation, and the level of income drawn each year from a pension. As a consequence, it is necessary to prepare to handle these changes in order to design appropriate life insurance and insurance products to be one of the sufficiency options. For example, financial products, called pension insurance or "annuity products," which are financial products that offer a guaranteed income stream, are used primarily by retirees. They help individuals to address the

risks of outliving their savings. Upon annuitization, the holding institution will issue a stream of payments at a later point in time. Therefore, annuity products or pension insurance are a tool to prevent risks from a longer life than a “longevity hedge”. This is like converting benefits for customers with the “cross subsidy” system using the Law of Large numbers. In this event, the risk will be shared among each other so that customers with long life expectancy have enough money based on that longevity (Longevity Risk Management). In the meantime, however, the Thai government urgently needs to create policies and/or measures to support the increasing number of Thai elderly in the near future that are appropriate and effective.

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